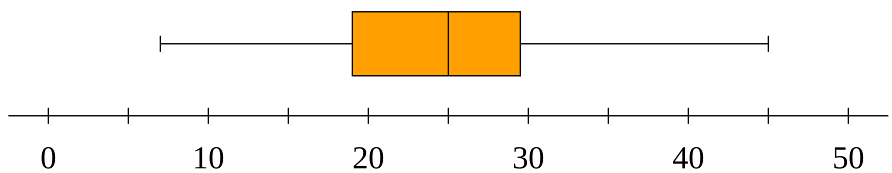
Math 121 **Test #2 Answers** **Spring 2009**

$(25, 47, 68, 84, 97)$, $\overline{x} = 65.0$.

1. (10 pts) $(3, 7, 7, 9.5, 10)$, $\overline{x} = 7.7$.

   (a) (6 pts) The boxplot:



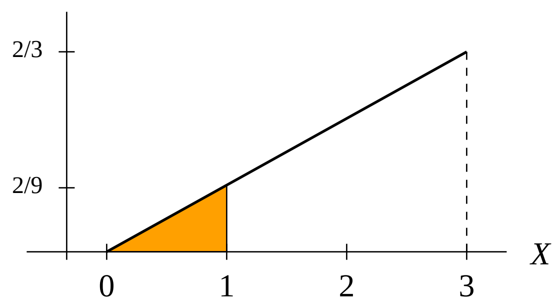
   (b) (4 pts) The shape appears to be *unimodal* and *skewed right*. That is because the intervals between the numbers get progressively larger as we go to the right, which means that the curve gets progressively lower.

2. (12 pts) $(8, 9, 11, 12, 12)$, $\overline{x} = 10.4$.

   Enter the data into the list `L₁` of the TI-83. Then select `1-Var Stats` and enter `L₁` and press `ENTER`. The statistics appear. The first one is $\overline{x} = 8.84$. Three lines below that is $s = 0.0503$. Scroll down and ways and find `Med=8.85` (the median).

3. (10 pts) $(0, 8, 10, 10, 10)$, $\overline{x} = 7.9$.

   The problem is to find the area under the graph between 0 and 1.



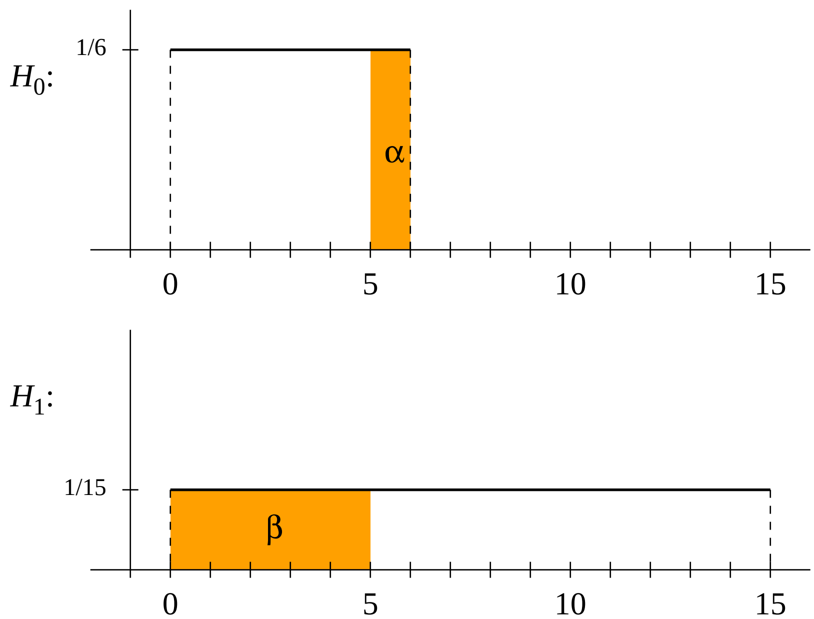
   The shape is a triangle with base $b = 1$ and height $h = \frac{2}{9}$ (one-third of the graph's height.) Therefore, the area is

$$\frac{1}{2}bh = \left(\frac{1}{2}\right)(1)\left(\frac{2}{9}\right) = \frac{1}{9}.$$

4. (18 pts) $(0, 9.5, 15, 18, 18)$, $\overline{x} = 13.1$.

   (a) (2 pts) The direction of extreme is to the right. That is clear from the graphs because the alternative graph extends much further to the right than does the null graph. Also, the decision rule says that we will reject H_0 if the observed value is 5 or *larger*.

   (b) (6 pts) In the null graph, draw a line at $x = 5$. The rejection region is to the right (from 5 to 6). Shade it and label it α. Its area is $\frac{1}{6}$. Therefore, $\alpha = \frac{1}{6}$.
   In the alternative graph, also draw a line at $x = 5$. The acceptance region is to the left (from 0 to 5). Shade it and label it β. Its area is $\frac{5}{15} = \frac{1}{3}$. Therefore, $\beta = \frac{1}{3}$.



   (c) (4 pts) Find the area under the null graph from 2 to 5. That area is $(5 - 2)\left(\frac{1}{6}\right) = \frac{1}{2}$.

   (d) (4 pts) Find the area under the alternative graph from 2 to 5. That area is $(5 - 2)\left(\frac{1}{15}\right) = \frac{1}{5}$.

   (e) (2 pts) The heights are set at the values needed to make the total area under the graph equal to 1.

5. (12 pts) $(0, 3, 9, 11.5, 12)$, $\overline{x} = 7.2$.

   For the first three, use `normalcdf` because you are finding a probability (i.e., area). On the second three use `invNorm` because you are finding a percentile.

   (a) $P(-1.5 < Z < 1.5) =$ `normalcdf(-1.5,1.5)` $= 0.8664$.

   (b) $P(Z > 2.5) =$ `normalcdf(2.5,E99)` $= 0.0062$.

(c) $P(82 < X < 120) =$ `normalcdf(82,120,100,15)` $= 0.7937$.

(d) The 15^{th} percentile of Z is `invNorm(.15)` $= -1.0364$.

(e) The 95^{th} percentile of X is `invNorm(.95,100,15)` $= 124.67$.

(f) To find the two values of X that determine the middle 20%, you first need to determine their percentile ranks. The middle 20% leaves the remaining 80%, half of which (40%) is to the left. Therefore, the first number is the 40^{th} percentile and the second number is the 60^{th} percentile. So the numbers are `invNorm(.40,100,15)` $= 96.2$ and `invNorm(.60,100,15)` $=$ 103.8.

6. (10 pts) $(0, 9, 10, 10, 10)$, $\overline{x} = 8.2$.

(a) (5 pts) Because cholesterol levels are normally distributed, we use `normalcdf` to get the answer. The probability is `normalcdf(-E99,175,190,20)` $=$ 0.2266.

(b) (5 pts) Because the problem asks for the approximate percentage, it is ok to use the "68-95-99.7" Rule. The approximate probability is 68%. However, it is easy enough to use the calculator. The probability is `normalcdf(-1,1)` $= 0.6827$.

7. (10 pts) $(0, 0, 2, 6, 10)$, $\overline{x} = 3.4$.

(a) (4 pts) The Central Limit Theorem says that the sampling distribution of $\hat{p}$ is normal, with mean p (which is 0.30) and standard deviation $\sqrt{\frac{p(1-p)}{n}}$ (which is $\sqrt{\frac{(0.30)(0.70)}{84}} = 0.05$). You could write simply that $\hat{p}$ is $N(0.30, 0.05)$.

(b) (6 pts) Using part (a), the probability is `normalcdf(.20,E99,.30,.05)` $=$ 0.9772.

8. (10 pts) $(0, 0, 6, 9, 10)$, $\overline{x} = 4.8$.

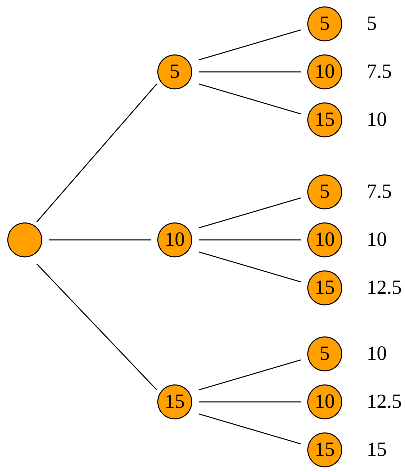
(a) (4 pts) The Central Limit Theorem says that the sampling distribution of $\overline{x}$ is normal, with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$. In this problem, $\mu = 2$, $\sigma = 0.7071$, and $n = 32$. Calculate $\frac{\sigma}{\sqrt{n}} = 0.125$. Thus, $\overline{x}$ is $N(2, 0.125)$.

(b) (6 pts) Using part (a), the probability that $\overline{x}$ is between 1.8 and 2.2 is `normalcdf(1.8,2.2,2,.125)` $= 0.8904$.

9. (8 pts) $(0, 0, 2, 4, 7)$, $\overline{x} = 2.3$.

A box contains a large number of vouchers. One-third of them are worth \$5, one-third are worth \$10, and one-third are worth \$15. Two vouchers are selected at random (with replacement). Let $\overline{x}$ represent the average value of the two selected vouchers. Find the sampling distribution of $\overline{x}$.

Draw a tree diagram of the possibilities. It should look something like the diagram below. I did not mark off if you did not draw the tree diagram; it is only a tool. However, I did give partial credit for drawing it if the final answer was wrong.



Now we can see what the possible averages are and in how many ways they may occur. For example, 10 can occur in 3 ways out of 9, so its probability of occuring is $\frac{3}{9}$. To get the sampling distribution of $\overline{x}$, summarize this in a chart.

$\overline{x}$	$P(\overline{x})$
5	1/9
7.5	2/9
10	3/9
12.5	2/9
15	1/9